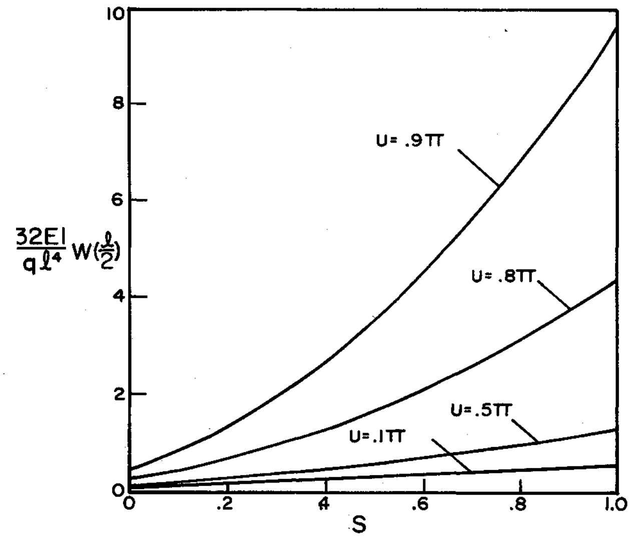

Fig. 1 Nondimensional deflection at $x = l/2$ for a simply supported beam column.

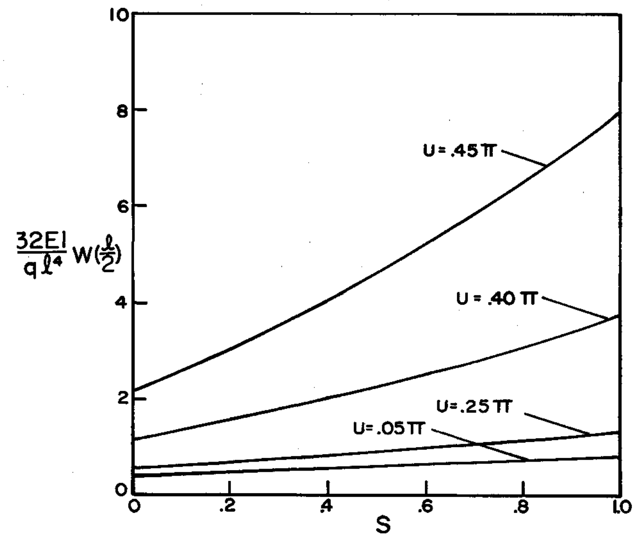

Fig. 2 Nondimensional deflection at $x = l/2$ for a clamped beam column.

$S^* \cong 0.10$, that is $E/G^* = 50$, is representative of some forms of pyrolytic graphite. However other new transversely isotropic materials may have an E/G^* ratio of 200 or larger,†† thus it is quite possible to talk meaningfully about a value of $S^* = 0.20$ or larger.‡‡

In conclusion, the effect of transverse isotropy (and beam geometry) as embodied in S (or S^*) is seen to significantly alter the solutions of the static problems solved. These effects are deleterious and hence must be considered when designing structures that utilize transversely isotropic materials. Finally, it is noted that these deleterious effects become more pronounced as the boundary restraints become more severe.

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†† This was learned in a recent conversation with the editor of *The Journal of Composite Materials*, S. Tsai.

‡‡ For example, if $E/G^* = 200$ and $h/l = 1/10$ $S^* \cong 0.20$.

Compressible Boundary-Layer Equations Solved by the Method of Parametric Differentiation

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Nomenclature

f = nondimensional stream function (as defined in Ref. 6)
g = $\partial f/\partial \beta$
g_1 = nonhomogeneous part of g defined in Eq. (7)
g_2 = homogeneous part of g defined in Eq. (7)
S = nondimensional enthalpy function (as defined in Ref. 6)
T = $\partial S/\partial \beta$
T_1 = nonhomogeneous part of T defined in Eq. (8)
T_2 = homogenous part of T defined in Eq. (8)
β = the pressure gradient parameter
η = the similarity variable
λ = $g''(0)$
μ = $T'(0)$
Subscripts
w = wall or surface value

Introduction

The method of parametric differentiation was developed by Rubbert[1] and Rubbert and Landahl[2] in connection with solving the transonic flow problems. This was a slight generalization of the concept of infinitesimal perturbation around a known solution originally given by Landahl.[3,4] Rubbert and Landahl[5] have applied this method for solving Falkner-Skan equations.

In this Note we illustrate this method by way of solving the compressible boundary-layer equations for Prandtl number unity. In the present problem β is the parameter for different values of which the solution is sought.

Equations and Solutions

Assuming such pressure gradients which give similarity solutions and the Prandtl number unity the boundary layer equations governing the flow of a compressible fluid become (Ref. 6, p 69)

$$f''' + ff'' + \beta(S - f'^2) = 0; \quad S'' + fS' = 0 \tag{1}$$

The boundary conditions are

$$f(0) = f'(0) = 0, \quad f(\infty) = 1 \tag{2a}$$

$$S(0) = S_w, \quad S(\infty) = 1 \tag{2b}$$

When $\beta \neq 0$ the Eqs. (1) together with the conditions (2) are a set of coupled equations required to be solved simultaneously. In the application of the present method the first step is to differentiate the Eqs. (1) and the boundary conditions (2) with respect to β to obtain

$$g''' + fg'' + f''g + \beta(T - 2f'g') + (S - f'^2) = 0 \tag{3a}$$

$$T'' + fT'' + gS' = 0 \tag{3b}$$

$$g(0) = g'(0) = g'(\infty) = T(0) = T(\infty) = 0 \tag{4}$$

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‡ Primes denote differentiation with respect to η.

where g and T are defined as

$$\partial f/\partial \beta = g \tag{5}$$

$$\partial S/\partial \beta = T \tag{6}$$

Equations (3) are a set of simultaneous linear ordinary differential equations for g and T. The functions f and S and their derivatives appear as coefficients. Once Eqs. (3) under the boundary conditions (4) are solved for g and T the required solutions for f and S can then be obtained from the first-order differential Eqs (5) and (6) either by a simple quadrature or using a Runge-Kutta procedure.

Equations (3) and (4) are solved by the following procedure of reducing the boundary value problem to an initial value problem. Let g and T be denoted by

$$g = g_1 + \lambda g_2 \tag{7}$$

$$T = T_1 + \mu T_2 \tag{8}$$

so that the required initial conditions $g''(0)$ and $T'(0)$ now become λ and μ, respectively. These are subsequently determined from the given conditions at the other end viz $g'(\infty) = 0$ and $T(\infty) = 0$

In view of the Eqs. (7) and (8) Eqs. (3) and (4) simplify to

$$g_1''' + fg_1'' + f''g_1 - 2\beta f'g_1' = f'^2 - S - \beta T \tag{9}$$

$$g_2''' + fg_2'' + f''g_2 - 2\beta f'g_2' = 0 \tag{10}$$

with

$$g_1(0) = g_1'(0) = g_1''(0) = 0, \tag{11}$$

$$g_2(0) = g_2'(0) = 0, \quad g_2''(0) = 1 \tag{12}$$

and

$$T_1'' + fT_1' = -gS' \tag{13}$$

$$T_2'' + fT_2' = 0 \tag{14}$$

with

$$T_1(0) = T_1'(0) = 0 \tag{15}$$

$$T_2(0) = 0, \quad T_2'(0) = 1 \tag{16}$$

For integration of Eqs. (9) to (16) the two-, three-, and four-point Falkner's predictor and Adam's corrector formulas[7] are employed. The step length in η for integration across the boundary layer is taken to be 0.1. and the integration is carried up to $\eta = 9$, taken to be the edge of the boundary layer.

In starting the solution of Eqs. (9) to (16) the Blasius value of $f''(0) = 0.4695999$ is assumed. Hence at $\beta = 0$, $S'(0)$ is also known. To obtain the solutions of g and T at a small step $\Delta\beta$, the values of functions $f(\eta)$, $S(\eta)$ and their derivatives at this β are required. The following iterative procedure was employed for this purpose.

As the distributions of $f(\eta)$ and $S(\eta)$ and their derivatives, are known at the previous β (to start with, for a given Sw, $\beta = 0$ and the solutions $f(\eta)$, $S(\eta)$ were obtained from Eqs. (1) and (2)), they are used as first approximations in Eqs. (9) to (16) for the current β and the solutions for $g(\eta)$ and $T(\eta)$ are obtained. These are then used to obtain $f(\eta)$ and $S(\eta)$ and their derivatives which serve as second approximations, again for use in Eqs. (9) to (16). This procedure is repeated till the final distributions for $f(\eta)$ and $S(\eta)$ settle down. In the present work the iteration is stopped when the sum

$$|f''_{(j+1)}(0) - f_j''(0)| + |S'_{(j+1)}(0) - S_j'(0)|,$$

where the subscripts j and $(j+1)$ denote the j th and $(j+1)$ th iteration, is less than 10^{-7}.

As the purpose of the present Note is to demostrate the effectiveness of the method for the solution of the boundary-layer Eqs. (1) and (2), the solutions are obtained only for a few representative values of Sw and β. The results are compared with those of Cohen and Reshotko[8] in the Table 1. These authors obtained their solutions by the method of successive approximations reducing the Eqs. (1) and (2) to a set of integral equations. The comparison between the present method and that of Cohen and Reshotko[8] shows that the methods give identical values correct to 3 decimal places. The authors of that reference have themselves mentioned that their work is correct to $\pm$ 0.0002.

Table 1 Wall shear and heat-transfer parameters

S_w	β	$f''(0)$ Present method	$f''(0)$ Cohen and Reshotko	$S'(0)$ Present method	$S'(0)$ Cohen and Reshotko
	−0.3	0.3178	0.3182	0.4261	0.4262
	−0.2	0.3874	...	0.4477	...
	−0.15	0.4125	...	0.4547	...
	−0.14	...	0.4166	...	0.4554
0.0	−0.1	0.4340	...	0.4605	...
	0.0	0.4696	0.4696	0.4696	0.4696
	0.5	0.5812	0.5806	0.4942	0.4948
	2.0	0.7387	0.7381	0.5206	0.5203
	−0.14	...	0.3841	...	0.359
	−0.125	0.3954	...	0.3617	...
	−0.100	0.4122	...	0.3650	...
	−0.075	0.4278	...	0.3680	...
	−0.050	0.4425	...	0.3708	...
0.2	−0.025	0.4564	...	0.3733	...
	0.0	0.4696	0.4696	0.3757	0.3757
	0.5	0.6546	0.6547	0.4036	0.4030
	1.5	0.8695	0.8689	0.4267	0.4261
	2.0	0.9483	0.9480	0.4334	0.4331

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Rayleigh Wave Effects in an Elastic Half-Space

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THE importance of the Rayleigh waves since first demonstrated by Lamb[1] is unquestioned in dynamic half-space problems. Many investigations following Lamb's work have, as a part of the study, looked for their effects. Asymptotic expressions for the Rayleigh phase have been obtained in some recent works.[2–4] As is well known, all these studies are carried out on the basis of the far-field assumption. The concluding remark by Rayleigh[5]

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